QUESTION 2005

1. Find $\dfrac{dy}{dx}$ when $y = x^x$.

See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 1.

2. If $A = \begin{bmatrix} -1 & 2 & 6 \\ 4 & 5 & -4 \\ -2 & 0 & 1 \end{bmatrix}$ and $A^T + 2B = 3I_3$, then find B, where I_3 denotes identity matrix of order 3.

See Topic: MATRICES, Short Answer Type Question No. 2.

3. Show that the mapping f(x) = cos x is neither one-to-one nor onto where $f : R \rightarrow R$.

See Topic: SET THEORY, Short Answer Type Question No. 1.

4. If $y = \cos(m \sin^{-1} x)$, then prove that $(1 - x^2)y_2 + m^2 y = xy_1$.

See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 3.

5. If $f : z \rightarrow z$ is defined by $f(x) = x^2$ and $g : z \rightarrow z$ is defined by $g(y) = 2y$, then find $(f \ o \ g)(-2)$ and $(f \ o \ g)(2)$, where z = set of all integers.

See Topic: SET THEORY, Short Answer Type Question No. 2.

6. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 3 & 3 \end{bmatrix}$, find AB.

See Topic: MATRICES, Short Answer Type Question No. 3.

7. If $A = \{x \in z : 0 \le x \le 10\}$, $B = \{x \in z : 5 \le x \le 15\}$ & $C = \{x \in z : x > 5\}$; then find A – B and $(B \cap C) - A$, where z = set of all integers.

See Topic: SET THEORY, Short Answer Type Question No. 3.

8. Integrate: $\int xe^x dx$.

See Topic: **INDEFINITE INTEGRATIONS, Short Answer Type Question No. 1.**

9. The co-ordinates of the point P(2, 3) change to (3, -1) due to shifting of origin. Find the position of the new origin.

See Topic: **TRANSFORMATION OF CO-ORDINATES, Short Answer Type Question No. 1.**

10. Evaluate: $\int_0^1 \dfrac{dx}{1+x^2}$.

See Topic: **DEFINITE INTEGRALS, Short Answer Type Question No. 1.**

11. Evaluate: $\lim\limits_{x \to 0}(\sin 3x / \sin 4x)$.

See Topic: **FUNCTION OF SEVERAL VARIABLES, Short Answer Type Question No. 1.**

12. Solve the equation $x^3 - 7x^2 + 36 = 0$, given that one of its roots is double of another.

See Topic: **POLYNOMIAL, Short Answer Type Question No. 2.**

13. Evaluate $\lim\limits_{x \to a} \dfrac{1-\cos(x-a)}{(x-a)^2}$

See Topic: **FUNCTION OF SEVERAL VARIABLES, Short Answer Type Question No. 2.**

14. Evaluate $\lim\limits_{x \to 0} \dfrac{x \log \sqrt{1+x}}{\sin^2 x}$

See Topic: **FUNCTION OF SEVERAL VARIABLES, Short Answer Type Question No. 3.**

15. Evaluate $\lim\limits_{x \to 0} \dfrac{\tan 2x - x}{3x - \sin x}$.

See Topic: **FUNCTION OF SEVERAL VARIABLES, Short Answer Type Question No. 4.**

16. Using Mean Value Theorem of Lagrange, find out the approximate value of $\sqrt{51}$.

See Topic: **EXPANSION OF FUNCTION, Short Answer Type Question No. 1.**

17. A function f is defined by $f(x) = \begin{cases} x-3 & when \ x \leq 0 \\ x^2 & when \ x > 0 \end{cases}$. Show continuity of f at x = 0.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 3.

18. Differentiate $\dfrac{x^3}{(1+x^3)}$ with respect to x^4.

See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 4.

19. Verify that the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

See Topic: MATRICES, Short Answer Type Question No. 4.

20. If $u = \tan^{-1}\left(\dfrac{(x+y)}{(\sqrt{x}+\sqrt{y})}\right)$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{4}\sin 2u$.

See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 3.

21. Find the maxima and minima of $x^3 - 6x^2 + 9x - 8$.

See Topic: MISCELLANEOUS, Short Answer Type Question No. 1.

22. Determine whether the function $f(x, y) = \dfrac{xy}{x^2 + y^2}$, if $(x, y) \neq (0, 0)$ and $f(x, y) = 0$, if $f(x, y) = (0, 0)$ is continuous at the origin.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Long Answer Type Question No. 4.

23. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$; find the equation whose roots are $\dfrac{1}{\alpha^2\beta^2}, \dfrac{1}{\beta^2\gamma^2}$ and $\dfrac{1}{\gamma^2\alpha^2}$.

See Topic: POLYNOMIAL, Short Answer Type Question No. 3.

24. Show that the equation $x^5 + x^3 - 2x^2 + x - 2 = 0$ has at least two imaginary roots.

See Topic: POLYNOMIAL, Short Answer Type Question No. 4.

25. If α, β, γ be the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$ find the equation whose roots are $1 + \dfrac{1}{\alpha}, 1 + \dfrac{1}{\beta}$ & $1 + \dfrac{1}{\gamma}$.

See Topic: POLYNOMIAL, Short Answer Type Question No. 5.

26. State Lagrange's .... for

27. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then find A^2 and show that $A^2 = A^{-1}$.

See Topic: MATRICES, Long Answer Type Question No. 3.

28. Reduce the .... it.

29. Evaluate: $\displaystyle\int \frac{\cos x\, dx}{(2 + \sin x)(3 + \sin x)}$.

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 2.

30. Evaluate $\displaystyle\int \tan^{-1}\left(\frac{2x}{1 + x^2}\right) dx$

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 3.

31. Evaluate $\displaystyle\int \frac{\sin 2x\, dx}{a \cos^2 x + b \sin^2 x}$

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 4.

32. Show that $\displaystyle\int_0^\pi \frac{x \tan x\, dx}{\sec x + \tan x} = \frac{\pi}{2}(\pi - 2)$.

See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 2.

33. Solve $x^3 - 9x + 28 = 0$ using Cardan's method.

See Topic: POLYNOMIAL, Short Answer Type Question No. 6.

34. Is the following system of linear equations solvable? If yes, solve it by using Cramer's rule:

$x + y + z = 7,\ x + 2y + 3z = 16,\ x + 3y + 4z = 22.$

See Topic: MATRICES, Long Answer Type Question No. 4.

35. Expand $e^{\sin x}$ by Maclaurin Theorem in an infinite series stating condition of convergence.

See Topic: EXPANSION OF FUNCTION, Short Answer Type Question No. 2.

36. Determine the nature of the conic represented by $\dfrac{8}{r} = 4 - 5\cos\theta$.

See Topic: POLAR EQUATIONS, Short Answer Type Question No. 1.

37. If by a .... that $a + b = a' + b'$.

38. Determine the angle through which the axes must be rotated so that the equation $lx + my + n = 0\ (m \neq 0)$ assumes the form $ay' + b = 0$.

See Topic: TRANSFORMATION OF CO-ORDINATES, Short Answer Type Question No. 2.